Медицинская Информатика
Medical Informatics

Олег Пьяных, PhD
opiany@gmail.com

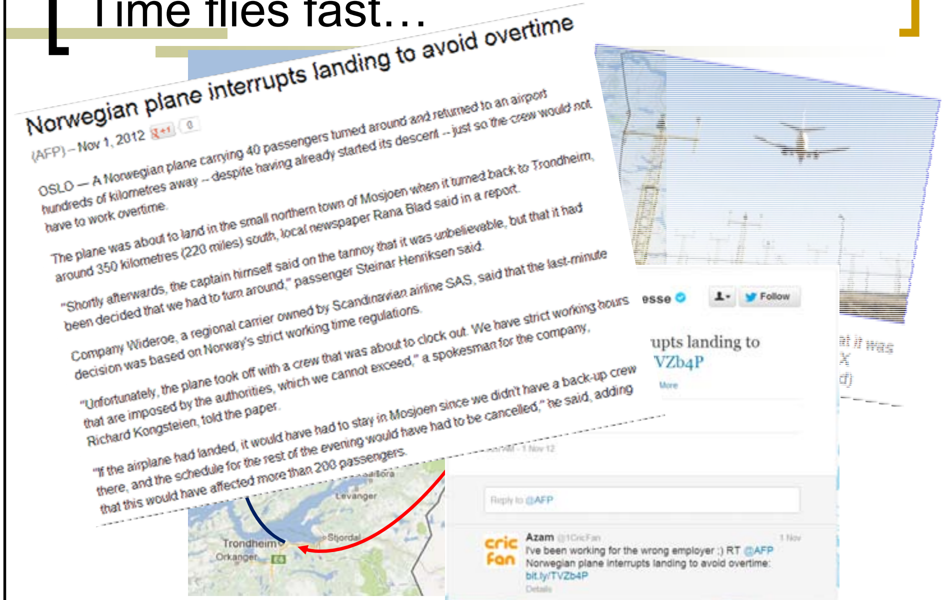
Математическое моделирование в медицине

Зачем?

- Математическое моделирование решает задачи расчета и оптимизации сложных процессов.
- Очень часто, создание модели является единственным способом анализа, ввиду сложности и случайности многих параметров процесса.
- Математическое моделирование крайне востребовано во многих областях медицинской практики, от исследования заболеваний до менеджмента.

Part 1: Scheduling

Time flies fast…



Norwegian plane interrupts landing to avoid overtime

(AFP) – Nov 1, 2012

OSLO — A Norwegian plane carrying 40 passengers turned around and returned to an airport hundreds of kilometres away -- despite having already started its descent -- just so the crew would not have to work overtime.

The plane was about to land in the small northern town of Mosjoen when it turned back to Trondheim, around 350 kilometres (220 miles) south, local newspaper Rana Blad said in a report.

"Shortly afterwards, the captain himself said on the tannoy that it was unbelievable, but that it had been decided that we had to turn around," passenger Steinar Henriksen said.

Company Wideroe, a regional carrier owned by Scandinavian airline SAS, said that the last-minute decision was based on Norway's strict working time regulations.

"Unfortunately, the plane took off with a crew that was about to clock out. We have strict working hours that are imposed by the authorities, which we cannot exceed," a spokesman for the company, Richard Kongsteien, told the paper.

"If the airplane had landed, it would have had to stay in Mosjoen since we didn't have a back-up crew there, and the schedule for the rest of the evening would have had to be cancelled," he said, adding that this would have affected more than 200 passengers.

Oleg Pianykh opiany@gmail.com

What is scheduling

- *Scheduling* is the process of deciding how to commit resources between a variety of possible tasks (Wiki).
- Scheduling typically involves *cost function (CF)* minimization under a set of constraints:
  - *Hard constraints (HC)*: To be satisfied exactly. Example: 11 nurses must be working in emergency department (ED) every day.
  - *Soft constraints (SC)*: To be satisfied within some limits. Example: nurses should have at least 3 days between night shifts.
  - Example of cost function: Find the optimal distribution of nurse shifts which meets specific patient flow.

Oleg Pianykh opiany@gmail.com

A bit of history

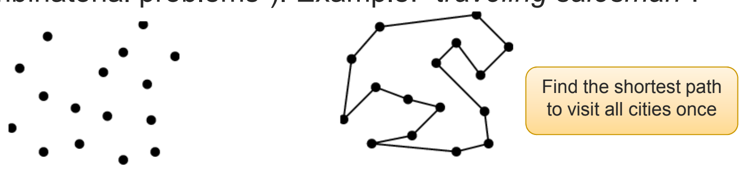
- Scheduling problems have been discussed for the past 40 years
- Have various applications in different industries: airplane crews (“team building”), post offices (optimal paths), health care, cargo shipments, etc.
- All industries differ, and their models and solutions cannot be easily translated across the industry boundaries.

Can’t I just do it by hand?

- Your HC, SC and CF can reflect very complex intermingled restrictions which cannot be done “the old way”, manually.
- The difference between suboptimal and optimal solutions can be significant, which makes search for strictly optimal solution(s) important.
- The risk of not meeting certain constraints can be too high (for instance, legislative requirements).

Don't forget about complexity

- Many scheduling problems are known to be NP-hard ("combinatorial problems"). Example: "*traveling salesman*":



- There are well-known algorithms (such as *simplex method* in *linear programming*) which can help. They are implemented in computational packages such as CPLEX.
- Solution existence is limited by data formats – such as integer constrains for people counts (cannot have 3.7 physicians).

Linear and Integer programming

- Linear programming: always finds the exact optimal solution (if this solution exists – well-posed problem)
- When problem does not fit the linear programming definition, sub-optimal approaches can be used: neural networks, genetic algorithms, simulated annealing. Sacrificing pure optimality to find an acceptable solution faster.
- Problem decomposition can help reduce complexity. Example: breaking annual scheduling in weeks – less optimal, but easier to solve.

Example 1: Simple linear programming problem

- Minimize shift cost given the following constraints for shift staffing:
  1. HC1: At least 2 fellows F (with salary C_F=1)
  2. HC2: At least one staff member S (with salary C_S=2)
  3. HC3: At most 4 physicians in total

Writing our equations

- What are the equations?

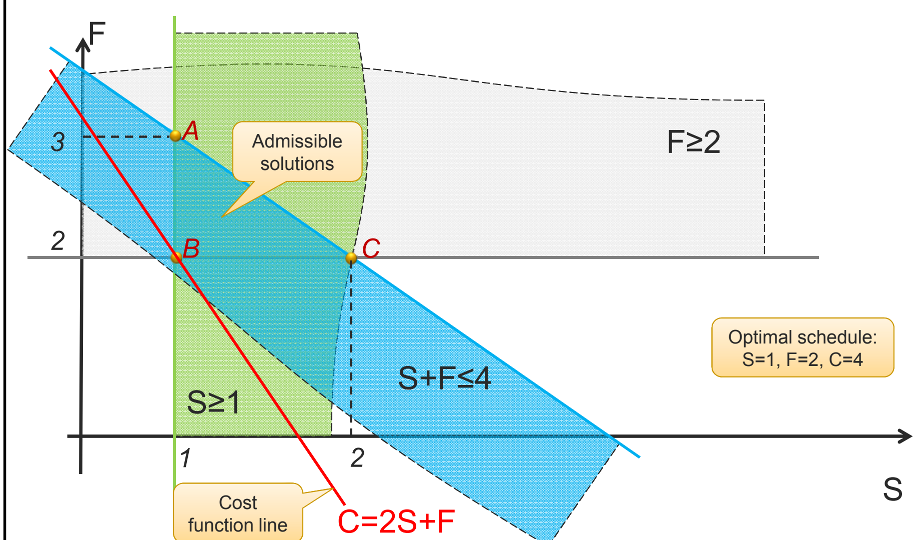
1. HC1: At least 2 fellows F (with salary C_F=1)
2. HC2: At least one staff member S (with salary C_S=2)
3. HC3: At most 4 physicians in total

Writing our equations

- Equations:
  1. Number of fellows F≥2
  2. Number of staff physicians S≥1
  3. Total number S+F≤4
- Cost function: $C = C_F{*}F + C_S{*}S \rightarrow \min$

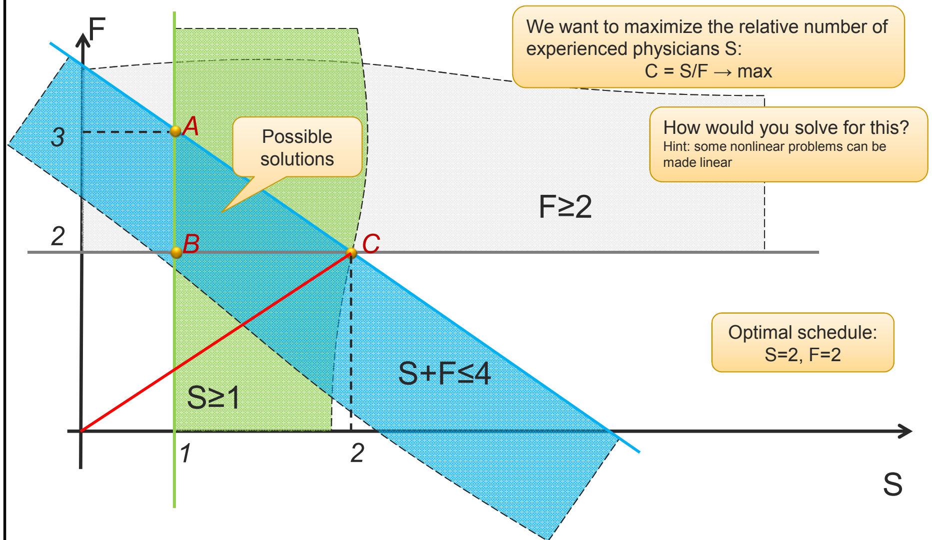
Oleg Pianykh opiany@gmail.com

Plotting our linear equations



Oleg Pianykh opiany@gmail.com

Nonlinear variation



Oleg Pianykh opiany@gmail.com

Example 2: Nurse scheduling

- Resource-allocation scheduling: assigning nurses to patients
- One of the earliest healthcare problems
- Number and choice of nurses are driven by the patient volume and flow, but
- Nurses need regular schedules (job satisfaction)

Oleg Pianykh opiany@gmail.com

Example 3: Physician scheduling

- Physicians are extremely expensive resources, and we need to optimize their allocation
- Physician work and shift assignments can vary depending on their skills and training, patient flow, work regulations/contracts, and many other factors.
- Physicians, just like any other employees, want to be satisfied and fairly treated. Objectivity and employee satisfaction is one of the major scheduling priorities.

See:
Brunner JO, Bard JF, Kolisch R., "Flexible shift scheduling of physicians", Health Care Manag Sci. 2009 Sep;12(3):285-305.

HCs

1. A shift must span a minimum length of time and can start at any time during a workday. The minimum shift length is 6 consecutive hours without a break, or 7 h including an hour-long break.
2. A shift can be extended up to a maximum of 12 consecutive hours without a break assignment, or up to 13 h including a hour-long break, as specified in the general labor contract.
3. After a shift service ends, a rest period of at least 12 h is required.
4. When breaks are required, each shift has to be assigned one. In that case, the minimum and maximum shift lengths are extended by the length of the break. A break may start only after a fixed number of periods into a shift and must end prior to a post-break workstretch of a fixed length.

SCs

1. Each physician has an individual contract which specifies his or her regular working hours per week. All working time exceeding that amount is overtime. If fewer hours are assigned than are specified in the contract, the difference is undertime, which is first used to compensate overtime assigned in the previous week and then used for training and research if any remains.

2. Each physician can be assigned a maximum number of on-call services per week, which must be separated by at least 1 day. The maximum number of on-call services per week is 1. The exclusion of two consecutive services (hard constraint no. 3) is included in this assumption except at the interface of two consecutive weeks.

3. Considering each physician individually, all shift starting times in his or her schedule for an arbitrary week should be contained in an individual time window of pre-defined length.

CF

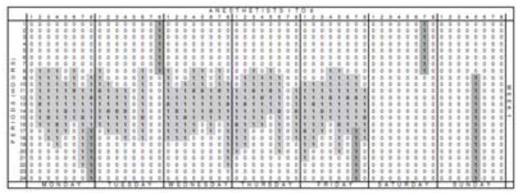
- We pay each physician his/her salary, which can also include different shift rates (such as overtime).
- We want to build a schedule with minimal total cost (total salary payments)

Oleg Pianykh opiany@gmail.com

Model, solution

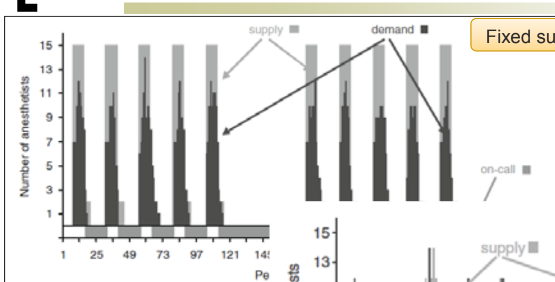
Model

Optimal roster

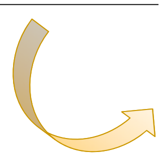


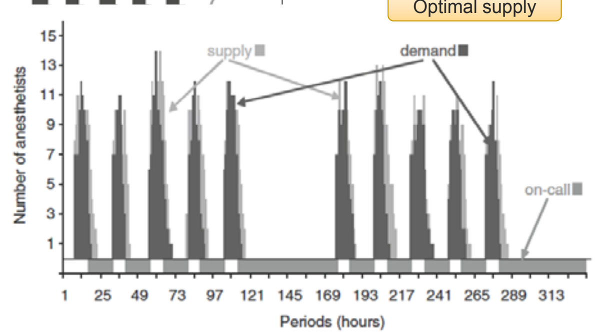
See:
Brunner JO, Bard JF, Kolisch R., “Flexible shift scheduling of physicians”, Health Care Manag Sci. 2009 Sep;12(3):285-305.

Gains in physicians’ supply



Fixed supply





Optimal supply

See:
Brunner JO, Bard JF, Kolisch R., “Flexible shift scheduling of physicians”, Health Care Manag Sci. 2009 Sep;12(3):285-305.

Advantages of math in scheduling

- Long-term scheduling under complex constraints
- Optimality, objectivity
- Finding optimal staffing and expense bounds
- Finding most satisfying schedules, most regular shifts

Disadvantages of math

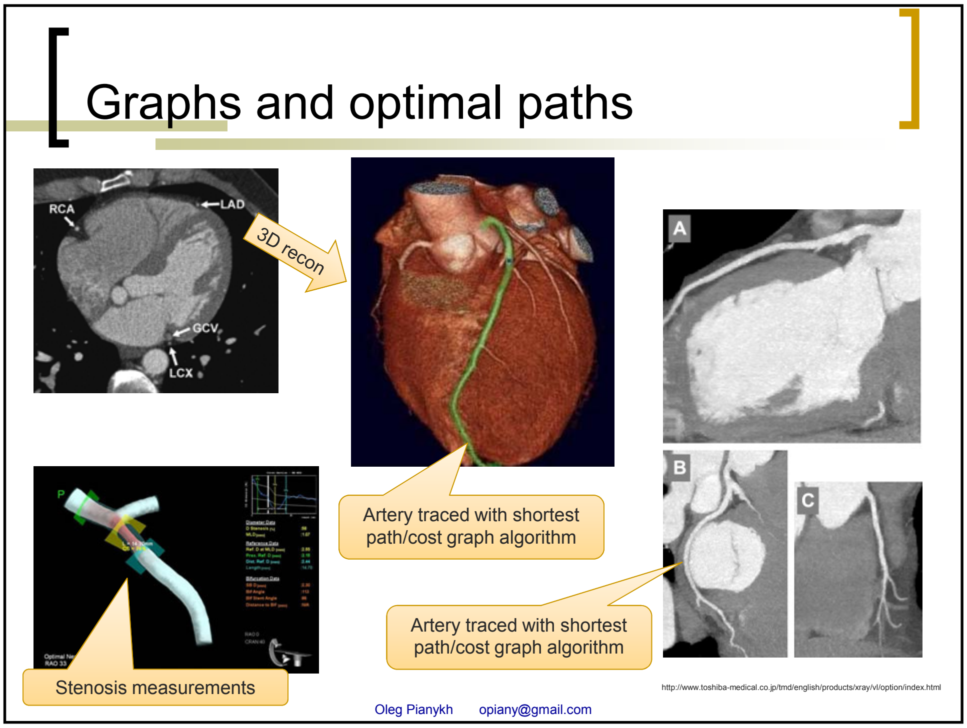
- Real life is hard to plan weeks ahead of time
- Complex models may take hours to compute (hard for "what if" analysis)
- Making models (and even defining the constraints) requires certain skill set

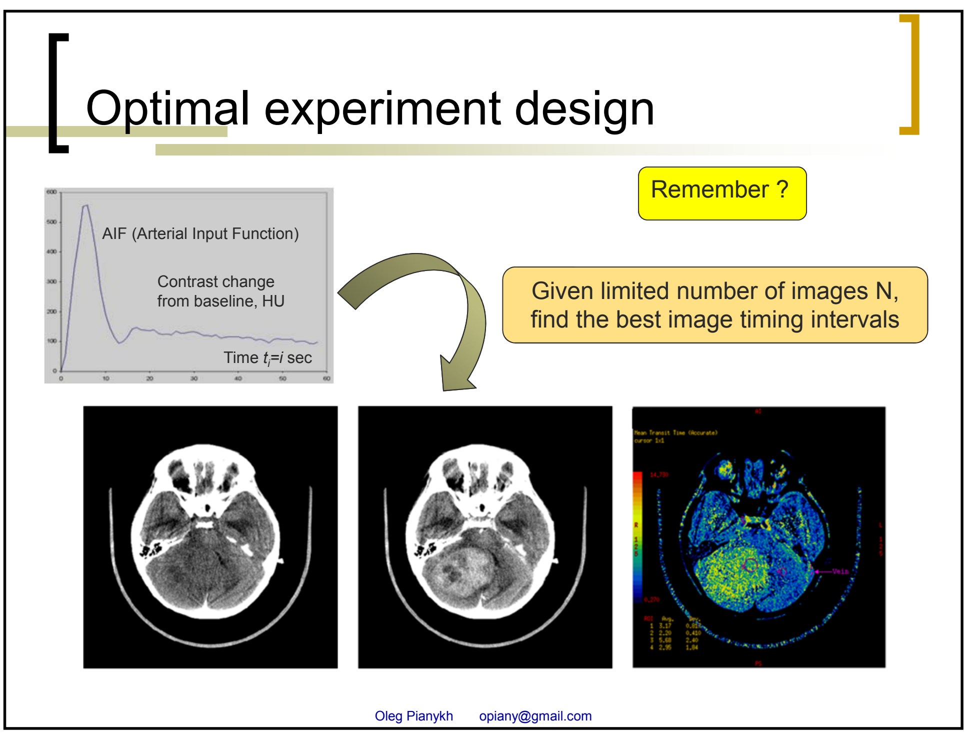
Oleg Pianykh opiany@gmail.com

Real life

- Implementing scheduling solutions can meet resistance:
  - “We use different rules”, “we do not want *your* rules”
  - “Too complex! (to learn/use/follow)”
  - Some departments and admins do *not* want to be objective, rational, and transparent
- Therefore, as always: *Start from the problem, not from the solution*
- Scheduling software must have: online access (mobile included), user right management, personal calendar integration, weekly statistics, simple interface

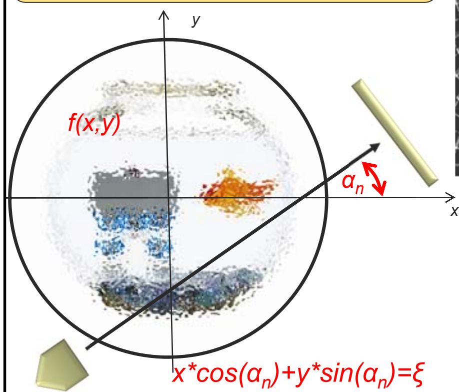
Examples of other optimization problems

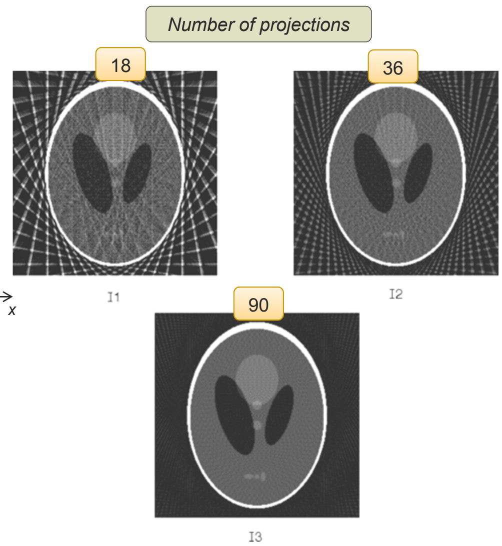

Graphs and optimal paths
RCA
LAD
GCV
LCX
3D recon
Artery traced with shortest path/cost graph algorithm
A
B
C
Artery traced with shortest path/cost graph algorithm
Stenosis measurements
http://www.toshiba-medical.co.jp/tmd/english/products/xray/vl/option/index.html
Oleg Pianykh opiany@gmail.com


Optimal experiment design
Remember ?
AIF (Arterial Input Function)
Contrast change from baseline, HU
Time t_i=i sec
Given limited number of images N, find the best image timing intervals
Oleg Pianykh opiany@gmail.com

Optimal experiment design

Given limited number of images N, find the best imaging angles α_n





Oleg Pianykh opiany@gmail.com

Part 2: Tracking systems

- Every model is meant to mimic a real process
- Any process is defined by its logic and parameters. If the logic is commonly known, the parameters need to be found from the real life.

Oleg Pianykh opiany@gmail.com

Tracking

- Tracking patients, doctors, resources

RTLS (Real Time Locating Systems)

- Only a few processes belong to coordinated workflow – the rest needs to be tracked
- See patient location in and out of the room, waiting time, which staff members and devices are in a room
- Track patient-staff interaction
- Track equipment (eliminate unproductive “hunting and gathering” tasks); track equipment states/conditions
- Real-time *alerts* on a variety of events (*threshold monitoring*) such as patient moving within and between departments, absence of equipment, long wait times
- Automatic temperature and humidity monitoring of refrigeration devices, warming devices, surgery rooms, and more

Often achieves the most impressive gains

Oleg Pianykh opiany@gmail.com

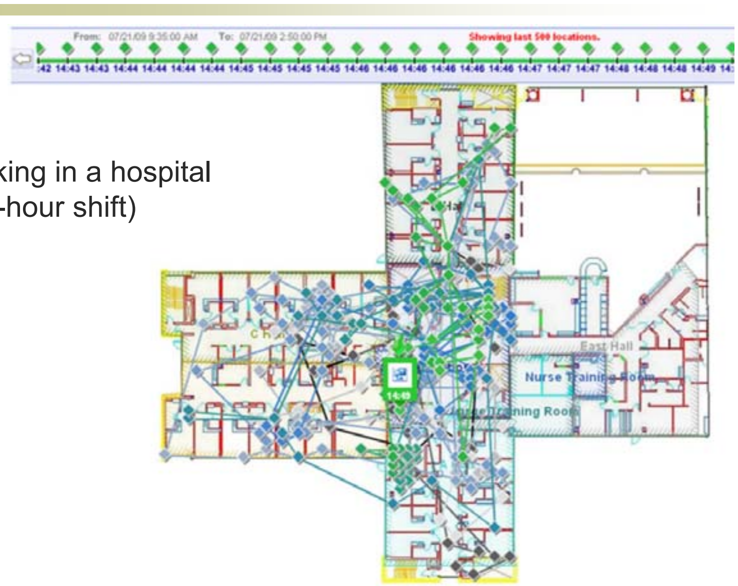
What (we hope) RTLS should deliver

- Increase staff efficiency and satisfaction by reducing manual data entry and improving staff communication
- Improve patient satisfaction through reduced wait times and real-time information to family members
- Augment existing patient tracking boards with real-time visibility information of patients, staff and equipment
- Decrease diversions by reducing wait times and identifying bottlenecks
- Increase revenue by identifying suboptimal workflows and improving patient throughput based on automated real-time visibility data
- Collect exhaustive historical data for further analysis and research

See http://www.aeroscout.com/patient-flow

RTLS in action

Nurse tracking in a hospital (over 6-hour shift)



RTLS in action

- Brief RTLS video: http://www.youtube.com/watch?v=KKyhc0IAZGA
- Watch later: http://www.youtube.com/watch?v=a4BHwLKcWYY&feature=related

Tracking providers

- www.radianse.com/products_overview.html: RFID
- www.sonitor.com/: Ultrasound
  - www.sonitor.com/show-me-how-patient-tracking (view demo)
- www.aeroscout.com/content/healthcare
  - www.aeroscout.com/flash/NewHC/start.html (view hospital demo)
- www.awarepoint.com, www.ekahau.com, www.centrak.com.

RTLS problems

- Low accuracy: for example, Wi-Fi (often preferred as the least expensive) has an average 30 feet error (=> good for equipment, but hardly for patients). Thick walls, signal interference can lower accuracy as well. “Asking colleagues” sometimes works better
- Wall-penetrating RTLS technologies (such as Wi-Fi) cannot be used to identify rooms, so second technology (such as ultrasound or infrared) is often added to provide room-level accuracy
- RTLS may need frequent calibrations
- Bulky RTLS tags with expiring batteries

RTLS: before you start using it

- Do staff surveys first – they do not cost anything, but can pinpoint existing problems
- Know your ROI. RTLS cost (initial and *maintenance*) may easily exceed savings on improvements
- Vicious circle: for optimal RTLS deployment you need to know the problem that RTLS has to solve

Part 3: Simulation

Why?

- We need models to process clinical data
  - Example: Data from tracking systems
- We need models to define healthcare policies
  - Example: Do we need to trace *N* contacts of a sick person to avoid flu pandemic, and what *N* should be?
- We need models for “what if” and “crisis” forecasts
  - Example: If we consider military intervention, how much humanitarian aid will be required for the refugees? Where the camps should be located (*geo simulation*) ?

Simulation as optimization

- Optimal problem examples in healthcare:
  - Optimal allocation of clinical resources (staff, beds, rooms, expensive equipment)
  - Optimal budget (e.g., minimal vaccination to prevent pandemic)
  - Optimal distribution (e.g., optimal vaccination targets to prevent pandemic)
- The main point of optimization: allocating maximum possible resource is not only expensive, but may not be the best solution.
  - Example: constant investments into epidemic control can result in diminished outcomes – it might be wiser to spend $$$ otherwise

Two modeling paradigms

- Discrete Event Simulation (DES)
  - System is modeled as a set of discrete (individual) agents, corresponding to different system resources (people, equipment, …)
  - The agents interact with each other according to their individual properties
- System Dynamics (SD)
  - We study the system as a continuous “mass”, evolving in time.
  - The evolution is described in state equations

System Dynamics (SD)

- Вместо индивидуальных черт системы, мы следим за ней как за единой массой, континуумом, меняющимся во времени
- Изменение описывается уравнениями состояния системы

SD Example: Predators and prey (P&p) model

- In a pond there live two types of fish: Predators ("Big fish" F) and prey ("small fish" s)
- Their population sizes at time t are defined by functions $F(t)$ and $s(t)$
- Population changes according to the following rules:
  - $F(t)$ grows proportionally to $s(t)$
  - $s(t)$ grows proportionally to $-F(t)$

P&p

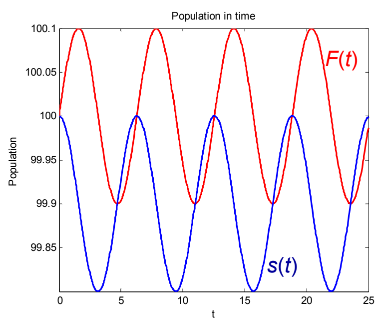
- We arrive to the following differential equations:

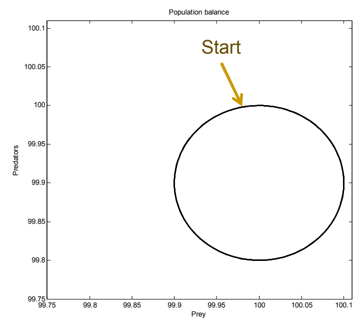
$$\frac{dF(t)}{dt} = s(t) + C_0, \quad \frac{ds(t)}{dt} = -F(t) + C_1$$

Let's use boundary conditions:

$$F(0) = s(0) = 100, \quad \frac{dF(t=0)}{dt} = 0.5, \quad \frac{ds(t=0)}{dt} = 0$$

- The solution shows population dynamics in time:



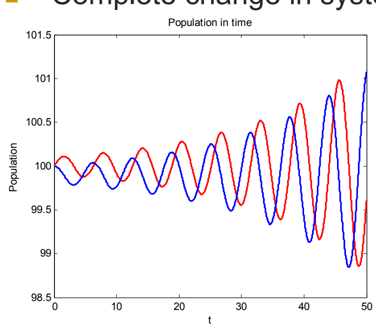


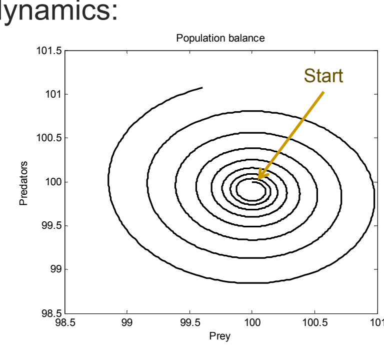
P&p

- Little modification in the first equation:

$$\frac{dF(t)}{dt} = 0.1F(t) + s(t) + C_0, \quad \frac{ds(t)}{dt} = -F(t) + C_1$$

- Complete change in system dynamics:



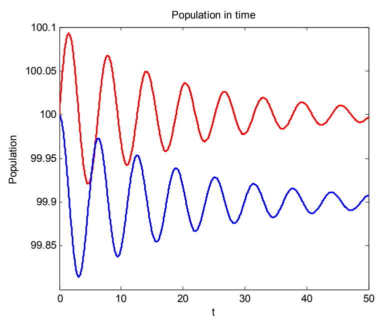


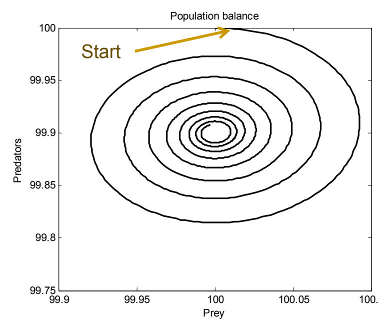
P&p

- Another little change in the first equation:

$$\frac{dF(t)}{dt} = -0.1F(t) + s(t) + C_0$$
$$\frac{ds(t)}{dt} = -F(t) + C_1 ,$$

- Complete transformation in system dynamics:





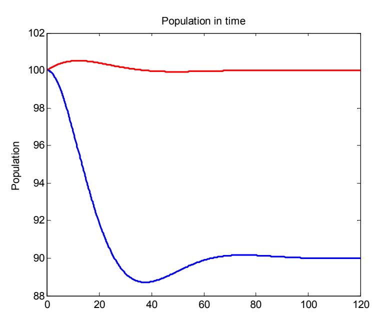
Oleg Pianykh opiany@gmail.com

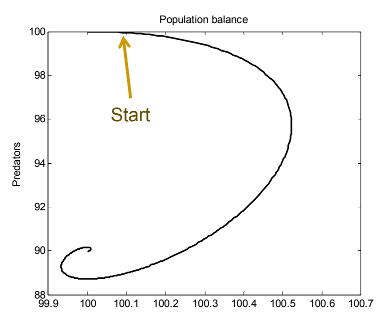
P&p

- Predators grow proportionally to prey *per predator*:

$$\frac{dF(t)}{dt} = -0.1F(t) + s(t)/F(t) + C_0$$
$$\frac{ds(t)}{dt} = -F(t) + C_1 ,$$

- Complete change in system dynamics:





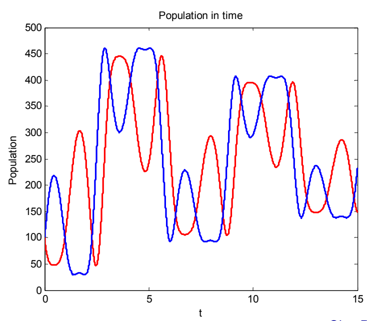
Oleg Pianykh opiany@gmail.com

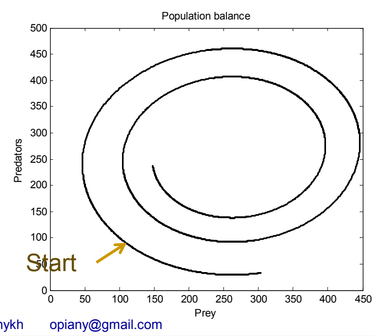
P&p

- Adding periodic environment (such as seasonal):

$$\frac{dF(t)}{dt} = (-0.1F(t) + s(t) + C_0)(1 + 3\cos(2t + 1)),$$
$$\frac{ds(t)}{dt} = (-F(t) + C_1)(1 + 3\cos(2t + 1))$$

- Significantly more complex and "seasonal" behavior:





P&p

- Small parameter changes can result in completely different outcomes (*butterfly effect*)

- P&p example in healthcare: infected (P) and healthy (p) people.

Discrete Event Simulation (DES)

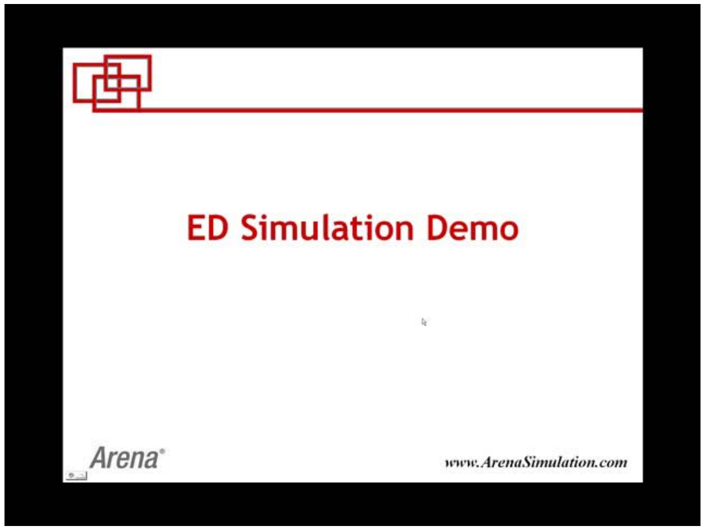
- DES: all agents (patients, staff, equipment, resources) are modeled as discrete entities, moving through different processing queues (e.g., treatment for patients)
- Ideal match for most real-life workflow modeling
- Easy to model, by assigning various properties to entities (patient age, sex, diagnosis, blood group, hair color, etc.)

Discrete Event Simulation (DES)

- Various statistical distributions and complex logic rules can be easily assigned to the processes
- Many user-friendly software packages available (Arena, MedModel, Simul8)
- Convincing model visualization: you can observe the entire process, at different stages, scenarios, and speed.

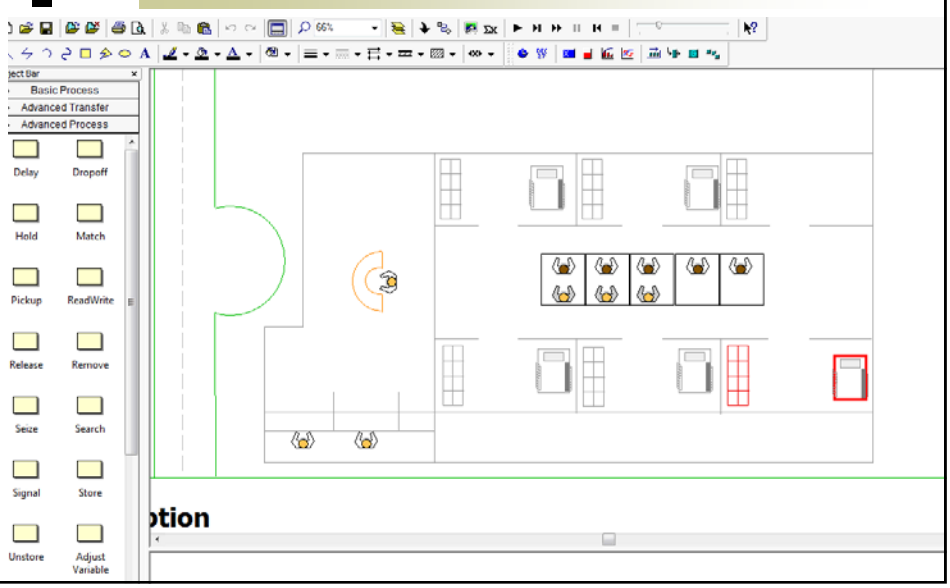
Oleg Pianykh opiany@gmail.com

Discrete Event Simulation (DES)



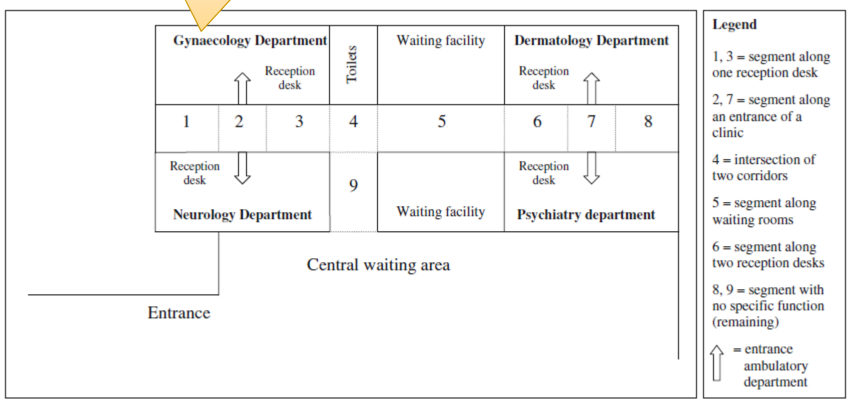
http://www.youtube.com/watch?v=U6I_HoB1E08

Emergency Room.doe (Arena)



Optimal hospital design

Different parts of the hospital can be optimized with DES simulation



Итог

- Современная клиническая практика часто сталкивается с задачами оптимизации
- Математическое моделирование позволяет вычислять наилучшие возможные решения,

  Но
- Перед началом любого моделирования оцените:
  - Оправданы ли расходы на него
  - Сможете ли вы воплотить в жизнь его результат
  - Не известны ли уже ответы на ваши вопросы.